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ABSTRACT

A linear regression model was used to select items from a pool of 700 arithmetic word problems to be used in a computer-assisted mathematics curriculum for elementary school students. The experimental procedure first involved a stepwise linear regression analysis of a student's performance over a set of 25 problems. The probability correct for each of the 700 problems was then predicted and the next 25 problems were selected for the student by matching predicted probability correct with the probability correct assigned to the student at the beginning of the study. The results showed a close match between the predicted and obtained probability correct when the experimental groups were evaluated. (Author/CH)

USING A LINEAR REGRESSION MODEL FOR ITEM SELECTION IN C.A.I.

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ABSTRACT

A linear regression model was used to select items from a pool of 700 arithmetic word problems for presentation in a computer assisted curriculum for elementary school students. The experimental procedure was:

1. Step-wise linear regression analysis of a student's performance over a set of 25 problems,
2. Prediction of probability correct (p(c)) for each of the 700 available problems, and
3. Selection of the student's next 25 problems by matching predicted p(c) with the p(c) value assigned to the student at the beginning of the study.

The results showed a close match between predicted p(c) and the obtained p(c) when the experimental groups were evaluated.

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Introduction

Computer Assisted Instruction (CAI) holds a constant promise of extending the range of individualization of instruction to levels not possible without the aid of the computer. One area in which that range can be extended is in tailoring curriculum to specific performance parameters of the student.

Most attempts to individualize instruction in CAI have consisted of simply altering the path of the student through a linear curriculum in one (or more) of three ways: skipping items, repeating items, or branching to items not included in the linear curriculum. More sophisticated approaches are possible. Several studies have been reported in which models of the student and/or the curriculum are used to build computationally complex strategies for molding the curriculum to data generated in the student-curriculum interaction. (see Fletcher, 1975 or Lorton & Killam, 1976 for discussions of these approaches.)

The research reported here concerns an attempt to devise a strategy for choosing, from a large pool, a unique set of exercises (arithmetic word problems) for each student. A regression model is used to characterize student performance on word problems, with structural features of the word problems serving as predictor variables. Major emphasis in the report is given to details of the procedure and to the relationship between properties of the exercises selected and the model characterizing the student. The report also compares success in predicting student performance using individual and group models, and examines the effect on progress through the curriculum and other performance parameters of assigning students to work at differing difficulty levels.

The Regression Model

Student performance is described using a regression model in which the independent variables characterize structural aspects of arithmetic word problems. The model is an adaptation of one for group data used in numerous previous studies at the Institute for Mathematical Studies in the Social Sciences (IMSSS) (Loftus & Suppes, 1972; Suppes, Loftus & Jerman, 1969). The regression model is:

$$p_{i,s} = a_{0s} + \sum_{j=1}^{k} a_{js} X_{ijs}$$

where $p_{i,s}$ takes the value .04 if the first response by student s to problem i is incorrect, .96 if the first response is correct. Definitions of the independent variables, X_j, are presented in Table 1, together with the range of each variable. The regression analysis was carried out using performance on a set of 25 word problems.

The PS Course

The experiment was conducted using a CAI course in solving arithmetic word problems. The course was developed at the Institute for Mathematical Studies in the Social Sciences (IMSSS), and is described in detail in Searle, Lorton and Suppes (1973). The student working at a computer terminal learns a set of simple commands for communicating with the instructional program. He solves word problems by instructing the computer to carry out the appropriate arithmetic operations. Thus, it is possible to investigate a student's problem-solving ability independent of his computational skill level.

Variable	Name	Range	Definition
X_1	OPERS	1-4	Number of different arithmetic operations required to reach a solution, using the coded solution string.
X_2	STEPS	1-9	Number of binary operations required to obtain an answer, using the coded solution string.
X_3	LENGT	7-79	Number of words in the problem. Each number symbol (#) counts as one word.
X_4	VCLUE	0, 1	Problem has a verbal clue (coded 0) if (1) operation is + and problem has word "together" or "altogether," or if (2) operation is - and problem has phrase "have left" or "were left," or if (3) operation is x and problem has word "each."
X_5	ORDER	0-3	The number of adjacent pairs of letters in the solution string that are not in alphabetical order.
X_6	ADD	0, 1	Solution requires an addition.
X_7	SUB	0, 1	Solution requires a subtraction.
X_8	MUL	0, 1	Solution requires a multiplication.
X_9	DIV	0, 1	Solution requires a division.
X_{10}	ALGER	0, 1	Problem statement is an algebraic statement, not a "story" (coded 1).

Table 1. Definition of Variables used in Regression Analysis.

The curriculum for the problem-solving (PS) course consists of a set of introductory problems (IPRBs) and 700 problems ordered by predicted difficulty level from easiest to hardest. The problem order was established for the course by using group performance data (Searle, Lorton and Suppes, 1973); that ordering is referred to in this paper as the Standard curriculum. The IPRB set contains 14 non-numerical problems that provide instruction in communicating with the computer and 25 numerical problems illustrating a variety of problem types.

Each problem in the PS course is coded to provide maximum flexibility for the curriculum driving program. Figure 1 is a sample section of the curriculum file for the PS course. Contained in the problem descriptions in this sample are examples of several key features of the program: generated values for the numbers within a problem, restrictions to which the generated values should conform, correct answers expressed as a mathematical expression, and explicit hints to be given to the student on request. Figure 2 shows some of the problems in Figure 1 as they would be presented to the student.

In transfering the problem text to the student, the program generates numbers at random to replace the # signs in the problem text. These numbers are generated to conform to the requirements in the problem specification labeled "[v:" and, using the information in "[s:" finds the correct answer to the problem which the student may type either as a symbolic expression or as a numerical value.

```
!E.401 v ()
        [How many pounds of beans can be packed in a box
that is # feet by # feet by # inches, if each pound
requires # cubic inches?]
        [v: :2.1 2.1 1 2.1]
        [s: 144xaxbxc/d]!

!E.402 v ()
        [How many pounds of fertilizer would be required to
cover a flower bed # inches by # inches if # pounds must be
applied to each 100 square feet?]
        [v: :2 2 2.1]
        [s: axb/144xc/d]!

!E.403 v ()
        [If a man can bind # sets of books in # days and
there are # books in each set, how many books does the man
bind in one day?]
        [v: :2 1 2]
        [s: a/bxc]!

!E.404 v ()
        [There were # telephones in Lake County in a recent
year. Assuming # people, and no more than one phone for each
person, what percent of the people had phones?]
        [v: b.gt.a]
        [s: a/bx100]!

!E.405 v ()
        [Mr. Larsen used # pounds of apples to fill baskets
with # pounds in each. He sold the baskets for # dollars
each. How much did he receive for his apples?]
        [v: a.em.b:3 2 1]
        [s: a/bxc]
        [h: Find out how many baskets there are.]!
```

Figure 1. Sample PS Course Problem File

```
-----------------
PROBLEM 401

HOW MANY POUNDS OF BEANS CAN BE PACKED IN A BOX
THAT IS  4.9 FEET BY  5.9 FEET BY 7 INCHES, IF EACH POUND
REQUIRES  3.7 CUBIC INCHES?
     A =       4.9
     B =       5.9
     C =       7
     D =       3.7

*(A*12)*B*\*\B'(B*12)
     E =     4163.0

*E*C
     F =    29141.3

*F/D=
     G =    7876.022
        WELL DONE
-----------------
PROBLEM 402

HOW MANY POUNDS OF FERTILIZER WOULD BE REQUIRED TO
COVER A FLOWER BED 50 INCHES BY 75 INCHES IF  8.5 POUNDS MUST BE
APPLIED TO EACH 100 SQUARE FEET?
     A =       50
     B =       75
     C =        8.5
     D =      100


*A*B/144
     E =       26.042

*E*C/100
     F =        2.214

*2.214=
     G =        2.214
        GREAT
-----------------
PROBLEM 403

IF A MAN CAN BIND 70 SETS OF BOOKS IN 7 DAYS AND
THERE ARE 13 BOOKS IN EACH SET, HOW MANY BOOKS DOES THE MAN
BIND IN ONE DAY?
     A =       70
     B =        7
     C =       13


*13*70/7=
     D =     130.000
        NICE GOING
```

Figure 2. Sample PS Course Lesson

Procedure

The full curriculum of IPRBs and 700 ordered problems was used. The experimental procedure was as follows.

1. The student worked through the IPRBs. When the student had worked the last problem in the set, the computer program presented again any of the 25 numerical problems for which the appropriate data had not been obtained, and then assigned the student to a one of three difficulty levels, 65 percent, 80 percent or 95 percent. Difficulty level assignments were made in round robin fashion so that experimental groups had equal numbers of students.

2. A regression equation was calculated, using the a step-wise linear regression model. Table 1 lists the variables used in the regression.

3. The regression equation was used to predict the probability correct for all 700 problems in the problem set. Then the problem set was reordered from easiest to hardest, using the predicted probabilities.

4. A new set of 25 problems was selected for the student by choosing those problems whose predicted probability correct, (p[c]) was closest to the difficulty level at which the student was assigned to work. Thus, for a student assigned to the 80 percent group, the computer program found the first problem in the ordered set for which p[c]-.80<0 and, choosing equally from above and below, selected 25 problems.

5. The 25 problem identifiers were stored in the student's history record and in subsequent lessons he worked these problems. When he completed the problem set the analysis described in steps 1-4 (except for the assignment of difficulty level) was repeated.

Subjects

The experimental subjects were fourth, fifth, and sixth grade students enrolled in the IMSSS elementary mathematics CAI drill-and-practice program. Students took lessons at a teletypewritter connected to the IMSSS PDP-10 computer through telephone lines. A student became eligible for the PS course when his average grade placement on the math drill-and-practice program reached 4.0. Thereafter, he received a PS lesson every fifth day. Thus, each student started the course at a different time of year, and worked a different number of sessions. Three hundred ninety-six students worked on the PS course; 271 were from schools for the deaf in several parts of the country, the remainder from a primarily black California elementary school. One hundred sixty-one students completed the IPRBs, and of these 38 completed one or more sets of 25 problems beyond the IPRBs.

REGRESSION ANALYSIS PROCEDURE

The core of the regression analysis technique used in this study is a version of the UCLA Biomed program BMD02R — Step-wise Linear Regression. The version used in this study has been modified to run on the IMSSS PDP-10 Timesharing system and was further adapted for use in this study so that the entire procedure described above was automatic. That is a program was run each day which determined which students had completed either the IPRB group of problems or their assigned group of 25 problems. As students who had completed a block of problems were identified, performance information was collected, a step-wise regression was run, the regression equation coefficients were determined and new problems were selected and assigned on the basis of the predicted p(c) from the analysis. Figure 3 illustrates the results of this procedure for one student through two iterations.

STUDENT 2325 assigned to 95 (#77)

Performance on Introductory Problem Set

I.101 C 48 11 27 1010000	I.102 C 16 11 21 1001000	I.103 C 21 11 16 1001000
I.104 C 26 12 25 1010000	I.105 C 28 11 34 0010000	I.106 C 46 11 25 1010000
I.107 C 41 11 18 1001000	I.108 C 38 11 17 1010000	I.109 C 35 12 20 1010000
I.110 C 23 11 17 1010000	I.111 C 23 11 48 0000100	I.112 C 30 11 17 1001000
I.113 X 66 11 19 0000100	I.114 C 18 11 23 1010000	I.115 C 36 13 24 1010000
I.116 C 41 22 45 1011000	I.117 C 63 22 24 0001100	I.118 X 63 11 23 1000010
I.119 C 46 22 25 0010100	I.120 X120 11 15 1100010	I.121 X203 12 74 0000100
I.122 C 35 12 33 1010000	I.123 C 63 11 20 1000010	I.124 C 66 23 37 1011000
I.125 C 51 11 18 1100010		

Step-Wise Linear Regression Analysis

Variable	Mean	S. D.	Variable	Mean	S. D.
1. p(c)	-.939	1.033	2. time	49.840	38.867
3. OPERS	1.160	.374	4. STEPS	1.440	.651
5. LENGT	26.600	13.137	6. VCLUE	.760	.436
7. ORDER	.080	.277	8. ADD	.520	.510
9. SUB.	.280	.458	10. MUL	.200	.408
11. DIV	.160	.374	12. ALGER	.000	.000

STEP-WISE REGRESSION SUMMARY TABLE: for 1. p(c) (constant = -.9820)

step num	variable ent	variable rem	multiple r	multiple rsq	increase in rsq	f value for del	last reg coefficnts
1 ADD	8		.4543	.2064	.2064	6.2400	-1.0491
2 SUB	9		.6211	.3858	.1794	6.7161	-.8584
3 LENGT	5		.6544	.4282	.0425	1.6376	.0145
4 STEPS	4		.6646	.4417	.0135	.5070	.1588
5 DIV	11		.6696	.4484	.0067	.2407	.4294
6 MUL	10		.6709	.4501	.0017	.0610	.2834
7 VCLUE	6		.6713	.4506	.0005	.0147	.1188

PROBLEMS for the NEXT SESSION

C.220	.950	A.077	.951	C.238	.950	A.076	.951	C.239	.950
A.075	.951	A.078	.949	C.250	.953	A.079	.949	C.249	.953
C.251	.949	C.248	.953	A.056	.948	A.074	.953	A.080	.948
A.073	.953	A.081	.948	A.072	.953	A.082	.948	A.049	.953
A.011	.947	C.247	.954	D.371	.946	C.246	.954	A.083	.946

average probability correct of NEW SET = .950

Figure 3. Step-wise Regression Analysis for a Sample Student

Student 2325 condition = 95

p(c) on current problems .9600

Performance on Assigned Problems

C.220 C 46 13 11 1010001	A.077 C 26 11 19 1001000	C.238 C 13 11 28 0101000
A.076 C 23 11 19 1001000	C.239 C 13 11 28 0101000	A.075 C 71 11 19 1001000
A.078 C 21 11 20 1001000	C.250 C 26 11 18 1101000	A.079 C 23 11 20 1001000
C.249 C 25 11 18 1101000	C.251 C 15 11 20 1101000	C.248 C 26 11 18 1101000
A.056 C 20 12 23 1010000	A.074 C 30 11 18 1001000	A.080 C 26 11 21 1001000
A.073 C 53 11 18 1001000	A.081 C 15 11 21 1001000	A.072 C 25 11 18 1001000
A.082 C 31 11 21 1001000	A.049 C 31 12 20 1010000	A.011 C 35 13 21 0010000
C.247 C 43 11 17 1101000	D.371 X281 12 11 1101001	C.246 C 60 11 17 1101000
A.083 C 25 11 22 1001000		

Step-Wise Linear Regression Analysis

Variable	Mean	S. D.	Variable	Mean	S. D.
1. p(c)	-1.270	.552	2. time	40.120	52.221
3. OPERS	1.000	.000	4. STEPS	1.280	.614
5. LENGT	19.440	3.787	6. VCLUE	.880	.332
7. ORDER	.360	.490	8. ADD	.160	.374
9. SUB	.840	.374	10. MUL	.000	.000
11. DIV	.000	.000	12. ALGER	.080	.277

STEP-WISE REGRESSION SUMMARY TABLE: for 1. p(c) (constant = -6.2015)

step num	variable	ent	rem	multiple r	rsq	increase in rsq	f value for del	last reg coefficnts
1	ALGER	12		.6922	.4791	.4791	22.0800	.7518
2	ADD	8		.7503	.5630	.0838	4.4083	-2.3021
3	STEPS	4		.7691	.5915	.0286	1.5385	1.6653
4	ORDER	7		.7766	.6031	.0116	.6176	.3192
5	VCLUE	6		.7806	.6093	.0062	.3175	1.1635
6	LENGT	5		.8121	.6595	.0502	2.8005	.1013

PROBLEMS for the NEXT SESSION

A.060	.949	D.359	.949	D.370	.951	A.082	.946	C.291	.955
C.248	.944	C.290	.955	C.249	.944	C.289	.955	C.250	.944
C.247	.955	C.294	.944	C.246	.955	C.295	.944	C.245	.955
C.296	.944	B.178	.957	C.297	.944	B.177	.957	A.021	.943
B.176	.957	D.372	.939	A.079	.957	F.559	.938	A.078	.957

average probability correct of NEW SET = .950

Figure 3. (con't) Step-wise Regression Analysis for Sample Student

RESULTS

In looking at the results we are concerned with several types of questions. The first group of questions concerns the feasibility of the experimental procedure and the characteristics of the resulting problem sets. The second group concerns the regression equations that arise during the experimental procedure, and lastly, we are interested in the success of the predictions.

1. Feasibility of the procedure and characteristics of the problem set.

The experimental procedure is illustrated using performance of a student on the IPRBs. The student worked 80 percent of the IPRBs correctly. The regression equation for the set of 25 problems was

$$z_i = -.90 - .23X_{i2} + .01X_{i3} - 1.61X_{i4} + 1.18X_{i6} + .67X_{i7} + 3.04X_{i9} \qquad (1)$$

with a multiple R of .69, a standard error of estimate of 1.09 and an R^2 of .48. Only 6 of the 10 variables entered the stepwise regression.

The cumulative frequency distribution of predicted probability correct obtained for the 700-problem set using equation 1 is shown in Table 2. For purposes of comparison the figure also shows the distribution for the Standard curriculum. The Standard curriculum was ordered using the regression equation

$$z_i = -1.92 + 1.38X_{i1} + .002X_{i3} + .18X_{i4} + .87X_{i5} - .37X_{i6} - .24X_{i7} + .47X_{i10}$$

13

Number of problems with $p < p(t)$

p(t)	Student	Cumulative	Standard	Cumulative
.00	134	134	209	209
.05	45	179	78	287
.10	47	226	41	328
.15	66	292	40	368
.20	71	363	73	441
.25	65	428	6	447
.30	43	471	12	459
.35	13	484	0	459
.40	17	501	15	474
.45	2	503	0	474
.50	2	505	0	474
.55	2	507	4	478
.60	3	510	6	484
.65	4	514	87	571
.70	5	519	1	572
.75	1	520	63	635
.80	3	523	35	670
.85	51	574	30	700
.90	18	592		
.95	108	700		

Table 2. Frequencies of problems at each target probability level

Thus, the variables used for predicting probability correct for the student were STEPS, LENGTH, VALUE, ADD, SUB, DIV; those used for predicting probability correct for the Standard curriculum were OPERS, LENGTH[1], VALUE, ORDER, ADD, SUB, and ALGER.

The set of 25 problems chosen for the student depended on his assigned difficulty level. The mean and range of p[c] for the set of problems that would have been selected at each level for the example student is shown in Table 3.

Difficulty Level	Mean p[c]	Range of p[c]
95	.949	.923 - .963
80	.787	.626 - .868
65	.639	.441 - .833

Table 3. Characteristics of Problem Set.

As can be seen in Table 2, fewer than 50 problems fall in the range .4 to .8 for the example student. Thus, the range of p[c] in the set of 25 problems is very wide for difficulty levels 65 and 80; for level 95 the range is quite narrow. The distribution of p[c]s was different for each student. However, in almost all cases the program

1 The variable, LENGTH, did not contribute significantly to the regression obtained from previous performance data but was used in constructing the Standard curriculum in order to provide a more varied mixture of problems. Without it large blocks of problems would have occurred requiring the same operation(s) for solution.)

was able to select a set of problems whose mean p[c] was very close to the assigned difficulty level. The mean difference for 214 cases was -.013, with a chi square value of 2.039 ($p < .001$) for the difference between mean p[c] and assigned difficulty level. These results indicate that the range of predicted probabilities for each student was sufficient to allow the selection of a set of problems at the assigned level.

The problem sets chosen for individual students showed marked differences from one another. For purposes of comparison, problems are numbered using their position in the Standard curriculum. The easiest problem in the Standard curriculum is problem 1, the hardest, problem 700. Table 4 shows the range of problems (using the Standard curriculum numbers) contained in the assigned set for four students at each difficulty level.

Difficulty Level 65	80	95
184 - 558	93 - 607	1 - 678
1 - 651	44 - 413	11 - 257
11 - 653	2 - 653	40 - 265
103 - 486	54 - 222	2 - 548

Table 4. Range of Problem Numbers in Assigned Set-Example Students

Condition		
65	80	95
271	457	118
334	208	112
361	263	249
188	100	367
114	196	73
310	194	157
293	237	66
329	304	118
43	343	236
233	215	200
124		142
		97
		156
		285
		241
		144
		85
		112
		62
		148

Table 5. Mean problem number for completed sets

Number of cases	Variable OPERS	STEPS	LENGT	VCLUE	ORDER	ADD	SUB	MUL	DIV
11		IN	IN	IN	IN	IN	IN	IN	
14	IN		IN	IN	IN	IN	IN	IN	
32	IN	IN		IN	IN	IN	IN	IN	
15	IN	IN	IN		IN	IN	IN	IN	
3			IN	IN	IN	IN	IN	IN	
2		IN			IN	IN	IN	IN	
1		IN	IN		IN	IN	IN	IN	
4	IN			IN	IN	IN	IN	IN	
2	IN		IN		IN	IN	IN	IN	
8	IN	IN			IN	IN	IN	IN	

Table 6. Pattern of Entry of Variables in the Regression (92 of 161 cases)

The ranges illustrated in Table 4 indicate that the p[c]s for the individual student were strikingly different from the p[c]s for the Standard curriculum. For example, consider the problem set at level 95 that contained problems 1 and 678 of the Standard curriculum. The mean predicted probability correct for that set (using the student's regression equation) was .923. The range of predicted probability correct for the same problems in the Standard curriculum (based on group data) was .887 - .001, with a mean of .276. (The student's performance on the set was .48.)

The mean problem number for 25 problems is shown for 41 students in Table 5. The distributions are presented for each difficulty level. The variability in the problems selected within each difficulty level shows clearly in the figure. The spread in mean problem numbers within each group is almost the same, and very wide.

Two conclusions emerge; the experimental procedure was able to produce a problem set at the desired predicted difficulty level for every student, and the degree of 'individualization' was indeed high, in that the uniquely determined problem sets differed greatly from each other. We turn next to a consideration of the regression equations that were used to construct these problem sets.

2. The regression equations

We'll look first at the regression equations characterizing performance on the 25 numerical IPRBs, which were completed by 161 students. The IPRB set was common for all students. It contained exemplars of 9 of the 10 variables; there were no ALGER problems in the

Performance on IPRBs
Variable

Coeff	CONST	OPERS	STEPS	LENGT	VCLUE	ORDER	ADD	SUB	MUL	DIV	ALGER
-5.00	2										
-4.50	6										
-4.00	8										
-3.50	13										
-3.00	23				8	12			3		
-2.50	22				5				3		
-2.00	30	1			3	4	4		2	1	
-1.50	19	2			1	34	9	2	3	1	
-1.00	14	13	10		2		29	7	2	8	
-.50	11	26	45	45	24	7	46	26	5	9	
.00	4	28	67	107	55	25	20	35	25	3	
.50	3	24	18		32		7	17	23	19	
1.00	2	21	7		8	35		1	29	20	
1.50	1	15	1		1	10	1	2	17	19	
2.00	1	9							8	20	
2.50	2					9			1	25	
3.00		1				1				1	

Table 7. Frequency Distribution for Regression Coefficients.

Mean Values

Trial	N	O	IP	O-IP	GP	O-GP
2	38	.603	.824	-.221	.576	.027
3	9	.564	.815	-.251	.551	.013
4	3	.547	.868	-.321	.487	.059
5	2	.470	.869	-.399	.484	-.014
6	1	.440	.941	-.501	.309	.131
7	1	.460	.917	-.457	.536	-.076

O = observed proportion correct for set of 25 problems
IP = individually predicted probability correct for set
GP = group predicted probability correct

Table 8. Comparison of Performance with Predictions.

set. The resulting regression equations contained from 4 to 8 variables. With 9 variables, 512 different patterns of entry and non-entry of variables into a regression equation are possible. The number of possible patterns with 4-8 variables is 372. The 161 equations were not distributed uniformly among the possible patterns; 52 different patterns occurred. Slightly fewer than half the equations (72 of the 161) conformed to one of four patterns, which are illustrated in Table 6. An additional 20 equations had the same configuration for the last five variables; these represent 5 additional patterns. Thus, performance of more than half the students was characterized by equations that included ORDER, ADD, SUB, and MUL, and did not include DIV.

The distribution of regression coefficients for 161 students for performance on the IPRBs is shown in Table 7.

3. Comparison of predicted and observed performance

Thirty-eight students completed at least one set of 25 problems (trial) beyond the IPRB set; a total of 54 student-trials were completed. Figure 4 shows a scatterplot of observed performance by condition for the 54 student-trials. Performance fell below predictions for 46 trials.

The mean observed performance for each trial beyond the IPRBs is shown in Table 8. The table also includes the mean predicted performance and the mean of the difference between predicted and observed. The large negative difference between observed and predicted for each trial reflects the data in the scatterplot. Table 8 also includes group prediction data. For each set of problems selected for a student, the mean predicted probability correct was calculated

Percent Correct	65	80	95
100			
96			1
92			1
88			
84	1		3
80	1	1	3
76			2
72			2
68			2
64	1		3
60		1	4
56	2	2	1
52		2	
48	2	3	1
44	1		3
40	1	5	
36	1		1
32			
28	1		
24		1	
20			
16			
12			
8	1		
4			

Condition

Figure 3. Scatterplot of observed performance by condition

using the predictions that had been used to construct the Standard curriculum. Thus, the 'group predicted probability correct' is the prediction for the set of problems using group data. As can be seen in the table, the group data were far more successful in predicting performance of individual students. For the 54 student trials Chi square for the difference between observed proportion correct and individual predicted probability correct was 60.31, that for the difference between observed proportion correct and group predicted probability correct was 4.82.

CONCLUSION

There is no question about the computational complexity nor the individualization possible through the process of selecting groups of problems for presentation to students based on a linear regression model. The procedure certainly was feasible and produced both an individualization and and analysis impossible with out the availability of a computer.

However, just as the experiment was a tour de force for the application of well defined models to CAI, it was also ambigious in the implication of the results to the instructional process. Draper & Smith (1966), in their discussion of model building and multiple regression, indicate that predictive models, such as that attempted here, can help pinpoint important variables and can be useful as variable screening devices. Certainly on the group level and functionally on the individual, the procedure used here performed those functions.

However, the procedure did not produce parameters that were particularly stable. Moreover, the variables entering into the equation tended to differ from student to student and from time to time. There remains much work to be done in order to be able to sample enough students to produce needed stability within the model. The equations produced seem to lack two of the important qualities Drapper & Smith mention - reasonable coefficients and plausible equations.

The instability can not be denied but it may be attributed to weakness in methods of measuring the criterion, the dependant variable, rather than some substantial insufficiency in the method itself.

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